CS 450: COMPUTER GRAPHICS

REVIEW:
MISCELLANEOUS MATH

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SETS AND CARTESIAN PRODUCT

• Sets
  • Represented by capital letters
  • a is a member of set S \( \rightarrow a \in S \)

• Cartesian product of two sets A and B \( \rightarrow A \times B \)
  • Creates new set \( \rightarrow \) all possible ordered pairs \((a, b)\) where:
    - Shorthand for \(A \times A \rightarrow A^2\)
    - Can be extended to more than two sets

\[ a \in A \]
\[ b \in B \]
## COMMON SETS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{R}$</td>
<td>The real numbers; 1-dimensional</td>
</tr>
<tr>
<td>$\mathbb{R}^+$</td>
<td>Non-negative real numbers (including zero); 1-dimensional</td>
</tr>
<tr>
<td>$\mathbb{Z}$</td>
<td>The integers; 1-dimensional</td>
</tr>
<tr>
<td>$\mathbb{R}^2$</td>
<td>Ordered pairs in real 2D plane; points in 2-dimensional Cartesian space</td>
</tr>
<tr>
<td>$\mathbb{R}^3$</td>
<td>Points in 3-dimensional Cartesian space</td>
</tr>
<tr>
<td>$\mathbb{R}^n$</td>
<td>Points in n-dimensional Cartesian space</td>
</tr>
<tr>
<td>$S^2$</td>
<td>Set of 3D points (points in $\mathbb{R}^3$) on the unit sphere; points are in 3D space BUT can be described with just 2 coordinates</td>
</tr>
</tbody>
</table>
MAPPING

• **Mappings or Functions**
  
  • **Programming:**
    
    • Input → argument of one **type**
    
    • Output → **returns** object of particular **type**
  
  • **Math (generally):**
    
    • Input → **member of** particular **set**
    
    • Output → **maps** input to **member of** a particular **set**
    
    • **Notation:** \( f : R \mapsto Z \)
DOMAIN, IMAGE, AND RANGE

• **Domain**
  • All possible input values

• **Image of a**
  • Given input member a from set A --> (image of a) = f(a) = d
  • Basically, point in output set

• **Range**
  • All possible output values
  • I.e., all images from every member of domain
INVERSE MAPPING

- Function f has an inverse function $f^{-1}$ IFF:
  - Every member in range of f $\rightarrow$ image of ONE and ONLY ONE point in domain of f
- Both function and inverse are then bijections
  - I.e., one-to-one mapping
    - E.g., $f(x) = x^3$
    - If a function is NOT a bijection, it has no inverse!
    - E.g., $f(x) = x^2$
    - Unless you restrict it to positive real numbers only $\rightarrow R^+$
INTERVALS

• **Open interval** → all values between a and b BUT NOT INCLUDING a and b
  • (a,b) → regular parentheses

• **Closed interval** → all values between a and b INCLUDING a and b itself
  • [a,b] → square brackets

• *Can be mixed:*
  • (a,b] → between a and b, including b BUT not including a
QUADRATIC EQUATION

- A quadratic equation has the form: \( Ax^2 + Bx + C = 0 \)
  - \( A, B, C \) = known constants

- Solving for \( x \) → finding points where parabola crosses x axis → finding “zero crossings” or “roots”

- Number of solutions depends on discriminant \( D \):
  - If \( D > 0 \) → two roots
    - Positive and negative square root
  - If \( D = 0 \) → one root
    - Square root of \( D \) is also zero
  - If \( D < 0 \) → no roots
    - Can’t take square root of a negative number and get a real number

\[
x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}
\]

\[
D = B^2 - 4AC
\]
ANGLES

- Angles can be expressed in **degrees** or **radians**
- If an angle is in **radians**:
  - Angle value = length of arc of unit circle “cut” by two lines of angle
- Convert to/from degrees and radians:

\[
\text{degrees} = \frac{180}{\pi} (\text{radians})
\]
\[
\text{radians} = \frac{\pi}{180} (\text{degrees})
\]
PYTHAGOREAN THEOREM AND TRIGONOMETRIC FUNCTIONS

• Given right triangle:
  • Longest side $\rightarrow$ the HYPOTENUSE $h$
  • For internal angle $\theta$:
    • Side OPPOSITE that angle $\rightarrow$ $o$
    • Side ADJACENT (next to) that angle (NOT hypotenuse) $\rightarrow$ $a$
  • Pythagorean theorem states: $a^2 + o^2 = h^2$

• Common trigonometric functions:
  \[
  \sin \theta = \frac{o}{h} \quad \cos \theta = \frac{a}{h} \quad \tan \theta = \frac{o}{a}
  \]
INVERSE TRIGONOMETRIC FUNCTIONS

• To get the angle back from sin/cos/tan value → asin, acos, and atan/atan2
  • NOTE: Should use atan2 instead of atan
    • Allows you to specify $o$ and $a$ separately → prevents division by zero
POLAR COORDINATES

• Instead of using x and y to define a coordinate → one can use polar coordinates:
  • \( \phi \) = COUNTER-CLOCKWISE angle from X axis
  • \( r \) = distance along line rotated by \( \phi \)

• Conversion from \((x, y)\) to polar:
  \[
  \phi = \text{atan2}(y, x) \\
  r = +\sqrt{x^2 + y^2}
  \]

• Conversion from polar to \((x, y)\):
  \[
  x = r \cos \phi \\
  y = r \sin \phi
  \]