MAT 115: Problem Set 4

Section: MW 4-5:50 pm

Due Date: 11/23/2015

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: /
Problem 1  Permutation and Cycles

A permutation is given in online-line, two-line or cycle form. Convert it to the other two forms. Give its inverse in all three forms.

(1) \((1, 3, 7, 8)(2, 5)(4)(6)\)

\[(2) \begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
8 & 3 & 7 & 2 & 6 & 4 & 5 & 1
\end{pmatrix}\]
Problem 2  Permutation Application:

Your friend tries to see if you are fast enough to catch him. He takes out four cups. He places a pea under the first cup. He quickly interchanges the cups in the second and third positions then the cups in the first and third positions and then the cups in the second and third positions. Finally he interchanges the second and fourth cup. The entire set of interchanges is done a total of five times.

(a) Write one entire set of interchanges as a permutation in cycle form:

(b) Write one entire set of interchanges as a permutation in adjacency matrix form:

(c) Where is the pea?
Problem 3  Permutation:

Let $f$ be a permutation of $n$. The cycle of $f$ that contains 1 is called the *the cycle generated by* 1. Please show the following:

(a) Prove that the number of permutations in which the cycle generated by 1 has length $n$ is $(n - 1)!$ (hint: you can try induction).
Problem 4  Permutation: the length of the cycle

All the permutations given below are in cycle form.
(a) Compute \((1, 2, 3)^{300}\)

(b) Compute \(((1, 3), (2, 5, 4))^{300}\)
Problem 5  Permutation: With Repetition

We work with the ordinary alphabet of 26-letters. Please solve the following:
(a) Define a 5-letter word to be any list of 5 letters that contains \textit{at least} one of the vowels A, E, I, O and U. How many 5-letter words are there?

(b) We can solve (a) in one single step but we can do it in 6 steps. Here are the steps:
(b-1) How many 5-letter words with exactly 1 vowel

(b-2) How many 5-letter words with exactly 2 vowels

(b-3) How many 5-letter words with exactly 3 vowels

(b-4) How many 5-letter words with exactly 4 vowels

(b-5) How many 5-letter words with exactly 5 vowels

(b-6) Your sum from b-1 till b-5 is? The result should be equivalent to (a)
Problem 6 Permutation: With NO Repetition

We work with the ordinary alphabet of 26-letters. Please solve the following:
(a) Define a 5-letter word to be any list of 5 letters that contains \textit{at least} one of the vowels A, E, I, O and U. How many 5-letter words are there?

(b) We can solve (a) in one single step but we can do it in 6 steps. Here are the steps:
(b-1) How many 5-letter words with exactly 1 vowel

(b-2) How many 5-letter words with exactly 2 vowels

(b-3) How many 5-letter words with exactly 3 vowels

(b-4) How many 5-letter words with exactly 4 vowels

(b-5) How many 5-letter words with exactly 5 vowels

(b-6) Your sum from b-1 till b-5 is? The result should be equivalent to (a)
Problem 7  Permutation: With and Without Repetition

We are interested in forming 3 letter words using the letters in LITTLEST. For the purpose of the problem, a word is any list of letters. Please answer the following:
(a) How many words can be made with no repeated letters?

(b) How many words can be made with unlimited repetition allowed?

(c) How many words can be made if repeats are allowed but no letter can be used more than it appears in LITTLEST?
Problem 8 Combinatorial:

You are in an Artificial Intelligence class and let suppose you design a robot. The task is given that your robot is set at coordinate (0,0) and your robot has to move to coordinate (9,12). Let say each time your robot can only move up by 1 in Y axis or right by one in the X axis. Extra conditions can be given, such as each move has different cost (eg. (1,1) to (1,2) costs 2 units but (1,1) to (2,1) cost 1 unit; (2,1) to (3,1) cost 5 units and (2,1) to (2,2) costs 1 unit and so on). As you can see, finding the optimal path (least expensive path) might be your task. However, to make your life easier, let us assume each move is of the same cost, therefore, all possible paths are of the same total cost. Please show how many paths there are for your robot to move from (0,0) to (9,12). [Maybe later when you take some optimization classes, you can help your robot to find the best route if each move has a different cost].