Problem 1 Power Set

Let $A = \{1, 3, 5, 6, 7\}$ and suppose $B$ is the power set of $A$, i.e. $B = \mathcal{P}(A)$.

(a) Please list the elements (subsets of $A$) in $B$.
(b) Let $C = \mathcal{P}(B)$. How many elements (subsets) are there in $C$?

Problem 2 Binomial Recursion

Please show

\[
\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}
\]

Problem 3 Induction Proof

Please prove the following by using induction proof. Make sure you mark the base case, hypothesis and the induction step clearly.

(a) Please show

\[
\sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}
\]

(b) Please show

\[
\sum_{i=1}^{n} (2i + 1) = n(n + 2)
\]

Problem 4 Rational Number

Show that $\sqrt{5} + 3$ is not rational, provided that we know $\sqrt{5}$ is not rational.
Problem 5 Other Proof Techniques: [Counterexample, Contrapositive, Contradiction, Case by case]

Prove the statement if true, otherwise find a counterexample. When you prove, if you see if and only if in the statement, you are supposed to prove for both directions.
(a) $\forall m, n \in \mathbb{Z}, m^3 - n^3$ is even if and only if $m - n$ is even.
(b) $\forall n \in \mathbb{Z}, n^2 - n + 2$ is even.
(c) For all distinct positive integers $m$ and $n$, both $m$ and $n$ are perfect squares if and only if $m + 2m^{1/2}n^{1/2} + n$ is a perfect square.
(d) For all distinct positive integers $m$ and $n$, both $m$ and $n$ are perfect squares if and only if $m^{1/2}n^{1/2}$ is an integer.

Problem 6 Practice Problems

For practice only. You do not have to turn in the solution.
Unit SF: 1.21
Unit NT: 1.2, 1.3, 1.4(b), 1.13, 1.14, 1.23, 1.27, 1.28